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Methods of Computational Physics

Homework Set #7

**Problem 1**

The Legendre Polynomials *y*, which are defined on the interval [-1, 1], satisfy the differential equation:

[1]

where is an eigenvalue I determine in this problem. Equation [1] admits even and odd functions, so I can reduce my analysis to the interval of [0,1]. I can get two boundary conditions from demanding regularity, giving:

At *x* *= 0*:

At *x = 1*:

Given that the equation is also linear and homogenous, the solutions can be normalized so that:

To solve this, I rewrote Equation [1] in first-order form, introducing as a new variable that satisfies the equation .

which you can solve independently, then bring together to integrate. This result indicates whether the current is correct (because if the result is 0, then Equation [1] is satisfied). Given that is the only varying quantity in the code, I used the root-finding method zbrent, which integrated the function from *x = 1* to *x = 0,* and evaluated the returned value of *y’* if the function was even, or *y* if the function was odd. Zbrent tries 3 point quadratic interpolation to locate root within a set of brackets. If interpolation fails, implying the root is outside the brackets, the method uses bisection. The function is then integrated over the interval [0,1], in N steps, which I set as 1000 for this project. To choose the brackets for the value of I made an educated guess based on Part C of this problem, where I guessed that would be a function of the number of roots, *l*, and would also dictate whether the problem was even or odd. My bracket equation was then:

My starting values were in terms of , where:

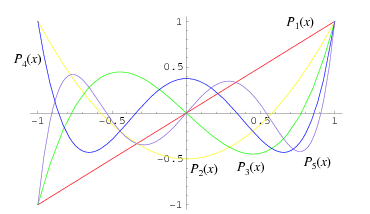
I got the following values for my first 5 values of

|  |  |  |
| --- | --- | --- |
| *l* | |  |
| 0 | 0 | |
| 1 | 2 | |
| 2 | 6 | |
| 3 | 12 | |
| 4 | 20 | |

`

This follows the pattern:

The graph of the polynomials is attached on the next page. It matches the analytical results from Wolfram-Alpha quite well, which I have included below:



*Weisstein, Eric W. "Legendre Polynomial." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/LegendrePolynomial.html*

